Final solns

Math 161 - 2022 Spring - Common Final Exam

Name: SOLNS

Section Number: ____________ **Instructor Name:** ____________________

Question	Points	Score
1	20	
2	8	
3	10	
4	10	
5	15	
6	10	
7	10	
8	15	
9	15	
10	10	
11	15	
12	12	
Total:	150	

- This exam has 12 questions worth a total of 150 points. Please check that your exam is complete, but otherwise do not look at the exam until the official start.
- Fill in your name and section above.
- Show your work. Correct work without corresponding work may not receive credit.
- You have 120 minutes to complete this exam.
- Technology of any kind is prohibited. The use of any notes is prohibited.

1. (20 points) Compute $\frac{\mathrm{d}}{\mathrm{d}x}[y]$ for:

(a) $y = (\tan x)e^{11x}$

$y' = (\tan x)' e^{11x} + \tan x (e^{11x})'$

$= \sec^2 x \, e^{11x} + \tan x (11 e^{11x})$

(b) $y = (x^7 - 1)^3 = u^3$

$y' = 3u^2 u'$

$= 3(x^7-1)^2 (x^7-1)'$

$= 3(x^7-1)^2 (7x^6)$

(c) $y = \cos(\sin(\pi x)) = \cos(u)$

$y' = -\sin(u) u'$

$= -\sin(\sin(\pi x))(\sin(\pi x))'$

$= -\sin(\sin(\pi x))(\cos(\pi x))\pi$

(d) $y = \dfrac{11x + 13 - \frac{17}{x}}{x^{2/3}}$. Then compute $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right]$

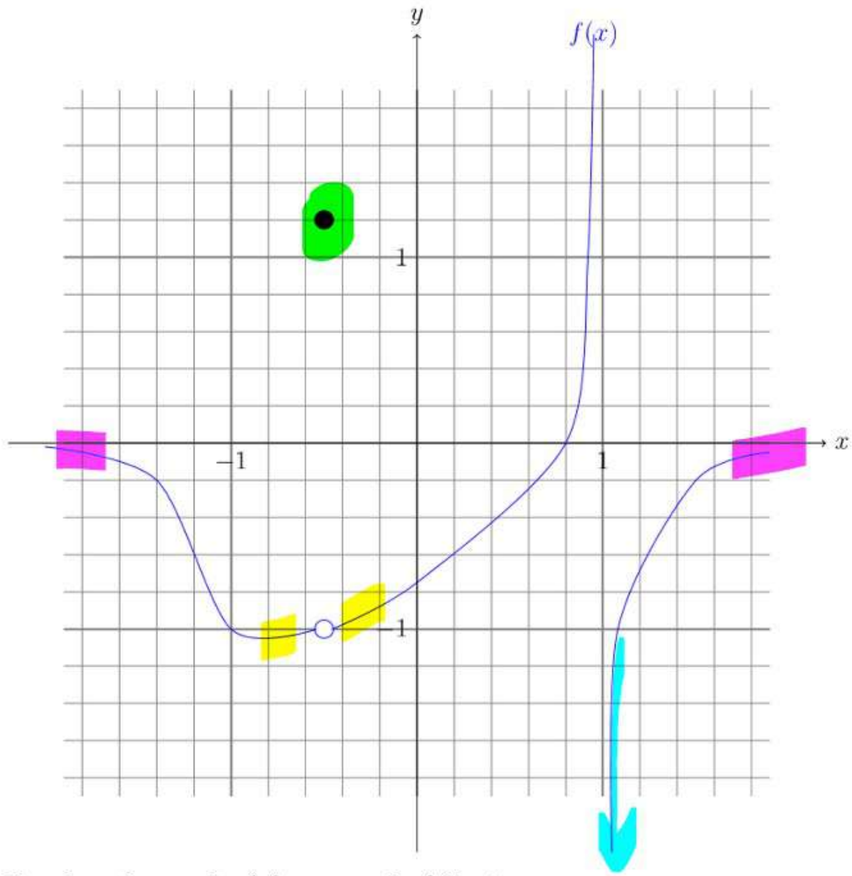
$y = 11x^{-1/3} + 13x^{-2/3} - 17x^{-5/3}$

$y' = 11\left(-\frac{1}{3}x^{-4/3}\right) + 13\left(-\frac{2}{3}x^{-5/3}\right) - 17\left(-\frac{5}{3}x^{-8/3}\right)$

$= -\frac{11}{3}x^{-4/3} + \frac{-26}{3}x^{-5/3} + \frac{85}{3}x^{-8/3}$

$y'' = -\frac{11}{3}\left(-\frac{4}{3}x^{-7/3}\right) + \left(-\frac{26}{3}\right)\left(-\frac{5}{3}x^{-8/3}\right) + \frac{85}{3}\left(-\frac{8}{3}x^{-11/3}\right)$

2. (8 points) The graph of the function $f(x)$ is shown below:



Based on the graph of f, answer the following:

(a) $\lim_{x \to -0.5} f(x) =$ -1

(b) $\lim_{x \to 1^+} f(x) =$ $-\infty$

(c) $\lim_{x \to \pm\infty} f(x) =$ 0

(d) $f(-0.5) =$ 1.2

3. (10 points) Consider

$$y = 3x^5 - 20x^3 - 75x + 999.$$

Find all critical points and all inflection points. You do **not** have to classify the critical points, but you **do** have to distinguish between potential inflection points and actual inflection points.

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Crit'l

$y' = 15x^4 - 60x^2 - 75$

$= 15(x^4 - 4x^2 - 5)$

$= 15(x^2 - 5)(x^2 + 1)$

$= 15(x + \sqrt{5})(x - \sqrt{5})(x^2 + 1)$

C.P. @ $x = \pm\sqrt{5}$

Infl

$y'' = 15(4x^3 - 8x)$

$y'' = 15(4x)(x^2 - 2)$

$y'' = 60x(x + \sqrt{2})(x - \sqrt{2})$

Potential I.P. @ $x = 0, \pm\sqrt{2}$

All potential inf. pts. are inf. pts.

as y'' changes sign across $0, -\sqrt{\ }, +\sqrt{\ }$

4. (10 points) Consider

$$y = \frac{x^2 + 6x + 9}{2x^2 - 18}.$$

Find all horizontal and vertical asymptotes. Classify any other discontinuities that exist.

H.A. $\lim_{x \to \infty} y = \lim_{x \to \infty} \dfrac{1 + \frac{6}{x} + \frac{9}{x^2}}{2 - \frac{18}{x^2}} = \dfrac{1}{2}$

V.A. Set denom $= 0$: $2x^2 - 18 = 0$

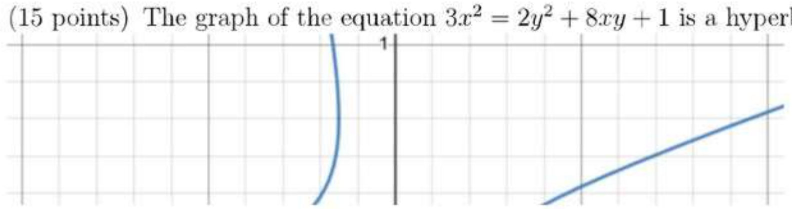
$x^2 - 9 = 0$

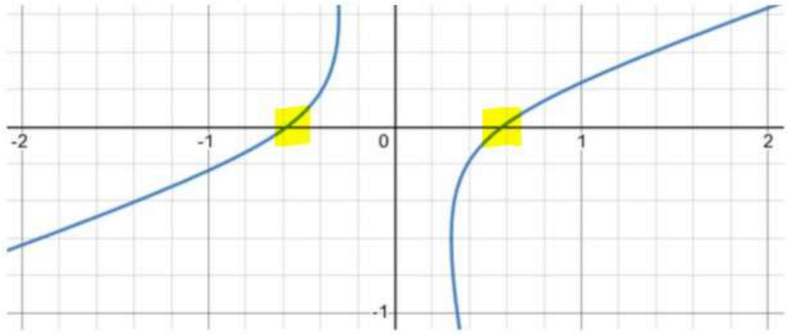
$x = \pm 3$

BUT $y = \dfrac{(x+3)^2}{2(x+3)(x-3)}$ so $x = -3$ is a remov. disc.

and $x = 3$ is only V.A.

5. (15 points) The graph of the equation $3x^2 = 2y^2 + 8xy + 1$ is a hyperbola as shown below:





(a) The hyperbola intersects the x-axis twice. Find the x values of those two points. (Eyeing this is not good, as they are not rational numbers.)

Set $y=0$ | $3x^2 = 1$ $\quad x = \pm\sqrt{\frac{1}{3}}$

(b) Implicit differentiation yields $6x = 4yy' + 8(y + xy')$. Solve for y'.

$$6x = 4yy' + 8y + 8xy'$$
$$6x - 8y = y'(4y + 8x)$$
$$y' = \frac{6x-8y}{4y+8x}$$

(c) Find the x values at which the tangent line is vertical.

Set $\frac{dy}{dx} = \frac{\text{something}}{\text{zero}}$ or $dx = 0$: $\quad 4y + 8x = 0$

when $x = -\frac{y}{2}$

Plug $\cancel{x = -\frac{y}{2}}$ $y = -2x$ into original fcn:

$$3x^2 = 2y^2 + 8xy + 1 \longrightarrow 3x^2 = 2(-2x)^2 + 8x(-2x) + 1$$
$$3x^2 = 8x^2 - 16x^2 + 1$$
$$11x^2 = 1 \quad \text{or} \quad x = \pm\sqrt{\frac{1}{11}}$$

6. (10 points) Find the point on the line $y = 3x - 2$ which is closest to the origin.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$d^2 = (x-0)^2 + (y-0)^2$$
$$d^2 = x^2 + (3x - 2)^2$$ [illegible]

$x = \frac{12}{20} = \frac{3}{5}$

Then $y = 3(\frac{3}{5}) - 2$

$= -\frac{1}{5}$

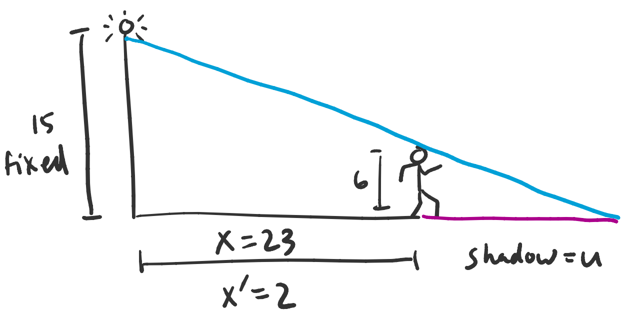
$$d^2 = (x-0)^2 + (y-0)^2$$
$$d^2 = x^2 + (3x-2)^2$$
$$= x^2 + 9x^2 - 12x + 4$$
$$= 10x^2 - 12x + 4$$
$$\frac{d}{dx}(d^2) = 20x - 12$$
$$0 = 20x - 12$$

Then $y = 3(\frac{3}{5}) - 2$
$= -\frac{1}{5}$

The point is $(3/5, -1/5)$

Finding $\frac{d}{dx}(d) = \frac{d}{dx}\left(10x^2 - 12x + 4\right)^{\frac{1}{2}}$ is also valid

7. (10 points) A streetlight is mounted at the top of a 15 foot pole. A 6 foot tall person walks away from the streetlight at 2 feet per second. How fast is the length of the person's shadow growing when the person is 23 feet from the pole?



Setup:

Similar $\triangle$'s:

$$\frac{u}{6} = \frac{u+x}{15}, \quad x = 23$$

$\frac{d}{dt}$ B.S.

$$\frac{u'}{6} = \frac{u'+x'}{15}$$
$$5u' = 2u' + 2x'$$
$$3u' = 2x'$$
$$u' = \frac{2(2)}{3} = \frac{4}{3}\ \frac{\text{ft}}{\text{sec}}$$

Units: u and x are in feet

u' and x' are $\frac{du}{dt}, \frac{dx}{dt}$,

so they are in ft/sec

8. (15 points) Consider the function
$$y = f(x) = e^{3x}.$$

(a) Find the equation of the line $L(x)$ which is tangent to $f(x)$ when $x = 2$.

$f(2) = e^6$
$f'(x) = 3e^{3x}$

Plug into

$y - y_1 = m(x - x_1)$

or

$y - e^6 = 3e^6(x-2)$

$f'(x) = 3e^{3x}$

$f'(x)\big|_{x=2} = 3e^6$ } into

or $y - e^6 = 3e^6(x-2)$

$L(x) = f(2) + f'(2)(x-2)$

$= e^6 + 3e^6(x-2)$

(b) Find the third-order Taylor polynomial centered at $c = 2$ for $f(x)$.

$$P_3(x) = f(2) + f'(2)(x-2) + f''(2)(x-2)^2 + f^{(3)}(2)(x-2)^3$$

Table

K	$f^{(k)}(x)$	$f^{(k)}(x)\big\|_{x=2}$	$k!$	a_k
0	e^{3x}	e^6	1	e^6
1	$3e^{3x}$	$3e^6$	1	$3e^6$
2	$9e^{3x}$	$9e^6$	2	$\frac{9}{2}e^6$
3	$27e^{3x}$	$27e^6$	6	$\frac{27}{6}e^6$

$$P_3(x) = e^6 + (3e^6)(x-2) + \left(\tfrac{9}{2}e^6\right)(x-2)^2 + \left(\tfrac{27}{6}e^6\right)(x-2)^3$$

9. (15 points) Evaluate the following limits

(a) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$ DE $\frac{4-4}{4-10+6} = \frac{0}{0}$ L'Ho

$= \lim_{x \to 2} \frac{2x}{2x-5}$ DE $\frac{4}{4-5} = \frac{4}{-1} = -4$

OR algebra: simplify to $\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} =$

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so $\lim_{x \to 2} \frac{x+2}{x-3} = \frac{4}{-1} = -4$

(b) $\lim_{x \to 1^+} \frac{e^x - e}{\ln x}$ $\underline{\text{DE}|}$ $\frac{e^1 - e}{\ln 1} = \frac{0}{0}$ L'Ho

$= \lim_{x \to 1^+} \frac{e^x}{1/x} = \lim_{x \to 1^+} x e^x$ $\underline{\text{DE}|}$ $1e^1$

OR $\underline{\text{DE}|}$ $\frac{e^1}{1/1} = e$

10. (10 points) Evaluate the following indefinite integrals:

(a) $\int x + 1 + \frac{1}{x} + \frac{1}{x^2}\,dx$

$= \frac{x^2}{2} + x + \ln|x| + \frac{x^{-1}}{-1} + C$

(b) $\int \frac{3x}{\sqrt{2+x^2}}\,dx$

$\left.\begin{aligned} u &= 2+x^2 \\ \frac{du}{dx} &= 2x \\ dx &= \frac{du}{2x} \end{aligned}\right\}$

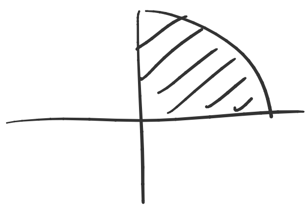
$= \int \frac{3x}{\sqrt{u}}\left(\frac{du}{2x}\right)$

$= \frac{3}{2}\int u^{-1/2}\,du = \frac{3}{2}\left(2u^{1/2}\right) + C$

$= 3(2+x^2)^{1/2} + C$

11. (15 points) Evaluate the following definite integrals:

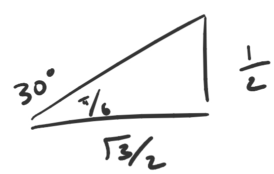
(a) $\int_0^5 \sqrt{25-x^2}\,dx$ Geometry: upper half circle of rad 5 from [0,5]



So $\int_0^5 \sqrt{25-x^2}\,dx = \frac{A_\odot}{4} = \frac{\pi(5)^2}{4} = \frac{25\pi}{4}$

(b) $\int_{\pi/6}^{\pi/2} 1+\sin(x)\,dx$

$= x - \cos x \Big]_{\pi/6}^{\pi/2}$



$= x \Big]_{\pi/6}^{\pi/2} - \cos x \Big]_{\pi/6}^{\pi/2}$

$= \left(\frac{\pi}{2} - \frac{\pi}{6}\right) - \left(\cancel{\cos\frac{\pi}{2}}^{\,0} - \cos\frac{\pi}{6}\right)$

$= \frac{\pi}{3} - \left(0 - \frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

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12. (12 points) Multiple choice

(a) Which function is an antiderivative of $f(x) = \ln(x)$?

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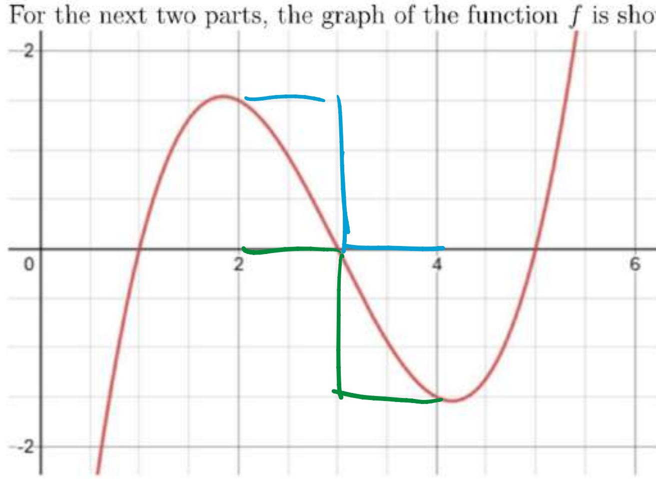
(a) Which function is an antiderivative of $f(x) = \ln(x)$?

(I) $\frac{1}{x}$ (II) $x\ln x - x$ (III) $x\ln x + x$ (IV) $\frac{(\ln x)^2}{2}$

(b) $\int_7^7 (e^{x^2}\cos(x))(x^5 - x^x)\,dx =$

(I) 0 (II) $(e^{t^2}\cos(t))(t^5 - t^t)$ (III) $(e^{t^2}\cos(t))(t^5 - t^t) + C$

For the next two parts, the graph of the function f is shown below:



(c) Using the graph above, which is the largest quantity?

(I) $\int_1^3 f(x)\,dx$ (II) $\int_3^5 f(x)\,dx$ (III) $\int_1^5 f(x)\,dx$

(d) Using the graph above, from $x = 2$ to $x = 4$, which is the largest quantity?

(I) L_2 (II) R_2 (III) T_2

would never be largest or smallest